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PLATO'S TRIANGLE AND GNOMONIC FACTOR: AN APPLICATION TO HERODOTUS' ORACLES

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ABSTRACT

A modification to the *gnomonic factor* (***fg***) (Perez-Enriquez 2000) using the concept of *Plato's triangle* is presented. With the aid of the platonic gnomonic factor (***fgp***) as we called it, we found that the oracles mentioned by Herodotus in his History (Dodona in Greece and Ammon in Oasis Siwa, Egypt) could have been placed there because the noon shadow of the sun of a vertical gnomon formed Plato's triangle the former, and the Egyptian "sacred triangle" (sides 3:4:5) the latter; sites where the value of ***fgp*** is equal to 1/1 and 1/3, respectively. This could mean that both concepts were known by Egyptians in Thebes long before, they were formalized by the Greeks. The priestesses, about whom Herodotus talks, knew the right angle triangle concept as an idealization of the sun's observation as it was proposed by Magdolen (2001), i.e., the triangle is the shadow casted by a gnomon. We consider that the idea of the use of a gnomon for site location could be found in other regions along the Valley of the Nile.

KEYWORDS: Dodona, Siwa Oasis, shadow, Egyptians, sacred triangle, methodology

1. INTRODUCTION

Egyptian Mathematics has not a well-defined beginning. There exist several hypotheses about how the new mathematics came, first in Mesopotamia and later in Egypt, and converted into the engineering that allowed the building of pyramids and necropolis (Smith, 1951). These great constructions of nearly 5000 years ago, implies the knowledge of geometry well above the level registered in the preserved documents of antiquity (papyrus and clay tablets) (Imhausen, 2006). On the other hand, the Astronomy practiced by these civilizations has been laid out in diverse media; among them, the painting and bas-reliefs of walls and ceiling of temples and buildings (Vazquez Hoys, 2012).

A fact that cannot be left aside is the role of the sun in these ancient cultures. Its main role was a god, the main one, represented by a solar disc or the rays coming out from it (Berggren, 2010). As a consequence, we consider that they must have had a very profound knowledge of his motion in the sky not only during daytime, but also, along the year. Surely, the observation of the sun allowed them to orient their pyramids (Kittler, 2008); and this must be expressed in some way in the Rind Mathematical Papyrus (RMP).

From these considerations about the importance of solar observations by ancient civilizations, the study of Stonehenge monument with the aid of a new concept, the *gnomonic factor* (*fg*) was proposed several years ago (Perez-Enriquez, 2000). The use of this factor, which can be summarized as the algebraic difference of the shadows cast by a gnomon (in winter and summer solstices) over the length of the gnomon (see Figure 1), has allowed us to interpret the Trilithons Horseshoe as a calendar of three seasons.

In Figure 1, the *fg* for the Stonehenge's Latitude is shown. In other Mesoamerican civilizations as the Toltec and Mayan, the factor has allowed to suggest an explanation for the locations of cities like Teotihuacan and Chichen-Itza and the 260 days calendar (Perez-Enriquez, 2001). Ruben Calvino, has used it for the interpretation of sacred altars "rehue" of the Patagonian civilization (2002).

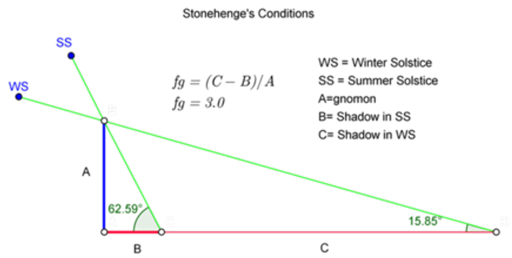

Figure 1 *Gnomonic factor* (*fg*) definition. A gnomon defines two right triangles at different times, from its shadows and a light ray coming from the sun; both shadows are related with the extreme positions of the sun along the year (solstices). The numbers appearing correspond to the Stonehenge case.

Now, we redefine such a concept in order to introduce the *platonic gnomonic factor* (*fgp*), with the use of the description of the Plato's Triangle that Xenophon Moussas introduced to us, recently (*pers. comm.*). Described, formally and apparently for the first time, by Plato in his Timeous (53b7-56e7) (Plato, 450 BC), this triangle shown in Figure 2, has sides $1:2:\sqrt{5}$ and an isosceles triangle overlapping it. This is mathematical object is also known as the "golden triangle" because from it can be found the "golden ratio" ($\Phi = \dfrac{1+\sqrt{5}}{2}$).

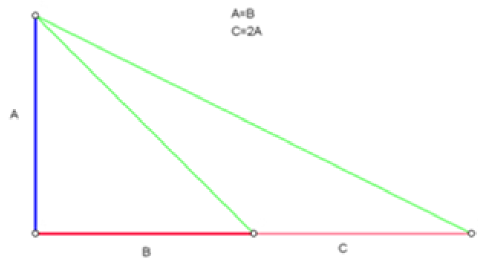

Figure 2. Plato's Triangle definition. It is formed by two right triangles: one isosceles and other with sides $1:2:\sqrt{5}$.

With this background, we begin the discussion of this paper. In section 2, we re-read the Herodotus' book *History*, the part related to the location of the oracles of Dodona, in Greece, and of Ammon, in the Siwa Oasis, Egypt: sites and origin. In the third section of this paper, we will continue with the concept of *platonic gnomonic factor*: combining the gnomonic factor and the Plato's Triangle concepts, in order to describe the methodology used in our analysis. From there, in section 4, we discuss how we have used it for the location of Dodona and Herodotus' oracle at Siwa Oasis. In the last section, we make some reflexions about our findings and their possible application of the method of the *platonic gnomonic factor* for the location of other cities and the orientations of buildings in the ancient world.

2. DODONA ORACLE'S ORIGIN ACCORDING TO HERODOTUS

In Herodotus *History* (440 BC), it is described the way Dodona and Ammon oracles were located. In Book II Euterpe (LIV-LVIII), he tells that two priestesses originally from Thebes, in Egypt, were responsible to tell the people of Greece, in the former, and from the Siwa Oasis, West Egypt in the latter, where the temples of divination had to be built. Also, he describes how Phoenicians carried the "black doves" from the south of Egypt to those places.

In effect, "that two of the sacred women were once carried off from Thebes by the Phoenicians, and that the story tells that one of them was sold into Libya, and the other in Greece, and these women were the first founders of the oracles in the two countries…", wrote Herodotus (*ibidem*).

From these indications and from the ruins that can be observed today, we can say that the Dodona's Oracle, erected to please Zeus, was located 22 km south from Janina, in Greece. The place of the "scared oak" can be set at 39°32'47" N Latitude and 20°47'16" E Longitude. In Figure 3, we show a *Google Earth* view of the site. The place selected according to the calculations, corresponds to the Theatre of the ancient theatre of Dodona as it can be found there.

Figure 3. Image of the area of the oracle of Dodona (Google Earth). We consider the centre of the forum as the point of observation of the Plato's Triangle.

On the other hand, we can observe the ruins of Ammon's Oracle, in the Siwa Oasis, located in the West side of Egypt. As it has been mentioned, Herodotus brings both oracles together with a common origin. This temple can be located in the following coordinates: Lat. 29°12'05" N, Lon. 25°30'59" E. In the central zone of Siwa Oasis there is a set of buildings on the top of Aghrumi Hill, corresponding to the the ruins of the oracle. As K.P. Kuhlmann mentions:

"...I understand Aghurmi Hill as place of the ancient Siwan acropolis (which includes the temple of their god Ammon) to have been a "natural" choice. It is my considered opinion that no "philosophical" or in any way "Greek" deliberations influenced the choice of place for the Temple of the Oracle. The latter is dated (by a royal name in Hieroglyphics) to the reign of Pharaoh Amasis (II; 570-526 BC)…" (Kuhlmann, *pers. comm.*). He made us the suggestion to look for a correspondence between dates with our calculations.

Herodotus heard about the possible origin of both oracles. What is the feature that both places have in common? What was the knowledge of these "sacred women" from Thebes that were sold as slaves? In the following sections we will try to give an answer to these questions.

Figure 4 Image area, where the ruins of the ancient Ammon's Temple are located in the Siwa Oasis (Google Earth). At this place the gnomon and its shadows formed the Egyptian "sacred triangle" (sides 3:4:5). The sun reached 45 degrees on November 9th.

3. THE PLATONIC GNOMONIC FACTOR

The *gnomonic factor* (*fg*) is a concept that was derived from the observations of the great trilithons of Stonehenge. Defined as the algebraic difference of the noontime shadows cast by a gnomon in the dates of extreme positions of the sun along the year (solstices) divided by the length of the gnomon (Perez-Enriquez, 2000, 2002), the *fg* results to be a good method to explore the cultures of our ancestors; we argue that this methodology of sun's observation was present in some way in the ancient world cultures which considered it as a god.

We can propose that their knowledge about the motion of the sun from south to north and from north to south, as the testimony of the Egyptian astronomer Harkebi from the XXX Dynasty (Bresciani, 2000), could have allowed them to identify it. Other civilizations like the Mesoamerican seem to have used this concept as it becomes apparent from the analysis of location of Chichen-Itza (Perez-Enriquez, 2001).

The definition itself of this factor implies the measure of the shadow of a vertical gnomon and, from there, the understanding of the right triangle. A good idea related to this matter was given by D. Magdolen in his paper about the astronomical origin of the Egyptian "sacred triangle" (Magdolen, 2001), "… At that time of the winter solstice, the shadow's length from the gnomon is recorded and compared to other seasonal culminating of the sun, but on this day ...[it] must have been one third longer than the proper gnomon's [length]...".

With the idea of Plato's Triangle, we have tried to find a place where it could be observed with a gnomon. There is no place on the planet with the solstices. However, if we use the winter solstice and the day when the sun is at 45°, we will find one: we introduce a modification to the *gnomonic factor* for defining a new one.

Definition. *The factor obtained by the division between the difference of length shadows observed in the Winter solstice (WS) and the day when the sun has a 45° elevation* (gnomon day *or **gd**) by the gnomon length (**g**) is called the* platonic gnomonic factor *(**fgp**) because it is inspired by the triangle shown in Figure 2.* (See Appendix 1)

$$fgp = \frac{wss - gds}{g} \quad (1)$$

Defined this way, we can ask: at what latitude did the ***fgp***=1 was observed when observing the noon solar shadows? Or in other words, in which place the sun had an altitude of 26.57° at its transit time of Winter solstice day?

The date of ***gd*** would be obtained as a result of the analysis as shown in Figure 5. In the following section, we describe how the *platonic gnomonic factor* could have been used to identify the sites where the oracles mentioned by Herodotus were located.

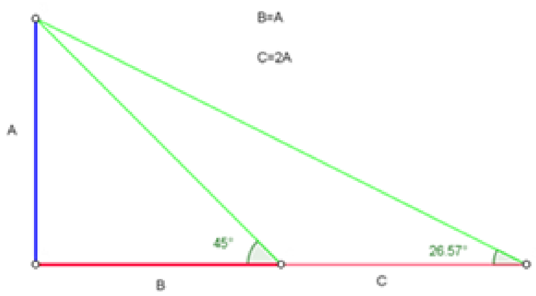

Figure 5. Sun's elevations correspond to the formation of Plato's Triangle. This triangle could have had been formed around 2500 years ago, with the solar light rays during the Winter Solstice and gnomon day (October 12th) in Dodona, Greece.

4. APPLICATION OF THE METHOD: THE HERODOTUS' ORACLES

Our first attempt to locate the place for Plato's Triangle Observation was to calculate, using a computer software program (for example, *Solar System Live* (Walker, 2012)), the positions of the sun for year AD 2010; i.e., the place where at Winter solstice the sun had an altitude of 26.57° at its transit and the corresponding date for 45° altitude also, a transit time.

We got a Latitude of 40.02° N as it can be seen in Table I; and for October 6th the sun reached an altitude of 44.84° there. With the aid of Google Earth (2012), we could locate the city of Janina in Greece as the place of this Plato's Triangle formation. Moreover, when making a search for the characteristics of Janina we discovered that not far from there, only 22 km to the South, the ruins of the first Greeks oracle can be found: the oracle of Dodona. This founding gave us a new perspective to our research.

TABLE I Sun's *Altitude to form the Plato's Triangle*[a]

Date	Angle	Year AD 2010 (Lat. 40.02° N)		Year 500 BC (Lat. 39.53° N)		Difference 2010 / -500
Winter Solstice (WS)	26.57	26.57	Dec. 22nd	26.70	Dec 26th	0.00 / 0.13
Gnomon Day (gd)	45.00	44.84	Oct. 6th	45.02	Oct. 12th	0.12 / 0.02
Summer Solstice (SS)				74.21	Jun. 26th	

[a], Angles in degrees.

The Dodona oracle, was mentioned by Herodotus as the one oracle founded by priestess coming from Thebes (Herodotus, 440 BC). Then, as it is shown in Table I, the solar altitudes were calculated for the coordinates (Latitude and Longitude) of the oracle of Dodona described in section 2. We have to make the remark that the solar positions were obtained for a date (year 500 BC) because it is near Herodotus' times. These results mean that the sun's observation at Dodona could represent the objective realization of Plato's Triangle and the ***fgp***=1/1 for Herodotus époque.

The successful application of the method described above and the coincidence that Herodotus had mentioned a second oracle of great importance in antiquity, lead us to the study of the triangle formed at that time (500 BC) at the Siwa Oasis. In Table II, it can be seen that transit sun's shadow at Winter solstice for the Ammon's temple site is 1.333 units when we consider a one-unit gnomon length. This result when expressed in gnomonic terms means that ***fgp***=1/3.

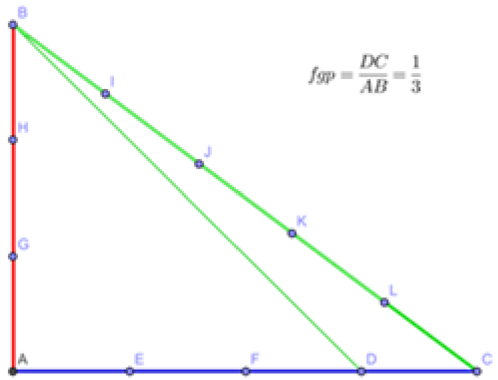

Figure 6. Triangle formed at the Ammon's Temple, in Siwa Oasis, about 4000 years ago. It is easy to verify that it is the Egyptian "sacred triangle" of sides 3:4:5.

In Figure 6, we can observe that the triangle formed by the gnomon and its shadows is nothing more that the Egyptian "sacred triangle" with sides 3:4:5, very well known as the Pythagorean triangle because it is the simplest triangle compatible with the condition of the theorem of Pythagoras.

Consequently, we consider that the two oracles described by Herodotus in *Euterpe* have at least three basic conditions in common: i) they were located according to the instructions given to the people by Theban priestesses; ii) the solar observation there formed triangles similar to Plato´s Triangle; and, iii) the value of the ***fgp*** is for both oracles a unit fraction (1/1 and 1/3, respectively). The former observation appears explicitly in Herodotus texts.

Regarding the second and the latter observations, which we can say that follow directly from this work, the places selected for the oracles could have been selected following directions coming from Thebes and they are not casual. In particular, the latter is related to the fact that 1 is reciprocal to itself and 1/3 is the reciprocal of 3.

Finally, we must establish the problem of precision: gnomonic observations would have been difficult but they would allow approaches to the reported values. As it can be seen in table II, the differences are of the order of tenths of length of the gnomon.

TABLE II *Herodotus' Ancient Oracles Analysis*

Herodotus' Site	Lat.	Lon.	Date	Height	Shadow	*fgp*
Oracle of Ammon, Siwa Oasis, Egypt	29.20° N	25.55° E	Dec. 26th, 500 BC	37.047	1.32478	
			Nov. 09th, 500 BC	45.158	0.99450	1/3
Oracle of Dodona, Greece	39.55° N	20.78° E	Dec. 22th, 500 BC	26.700	1.98828	
			Oct. 06rd, 500 BC	45.016	0.99944	1/1

[a], Height of sun for winter solstice and gnomon day.
[b], using a gnomon of unit length
[c], *platonic gnomonic factor* (See text)

5. CONCLUSIONS

The *platonic gnomonic factor*, has been presented as a tool for the study of location of cities, and as a possible method, applied by ancient cultures which had knowledge of the sun's motion in the sky along the year. In particular, we have shown that Plato's Triangle and the Egyptian "sacred triangle" could have been used, in ancient times, for the locations of the oracles mentioned by Herodotus in his book *History* (the oracle of Dodona and the temple of Ammon in West Egypt). Because the triangles can both be derived form the observation of the sun using a vertical gnomon, we can identify them as having a *fgp*'s value in terms of unit fractions: 1/1 for the former; 1/3 for the latter. Also, because Herodotus indicates that the oracles were built, probably, due to the suggestions of priestesses coming from Thebes, we can suggest that both triangles were known in Egypt and the priestesses were in search of sites where one could observe them. In some way, we think it is necessary to reread Herodotus and other classics in order to find in their texts such mathematical knowledge; look for them in the Rhind Mathematical Papyrus (RMP) or in the location of the cities along the valley of the Nile River.

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APPENDIX I On the Analytic Calculation of Platonic Gnomonic Factor (*fgp*)

The apparent extreme positions of the sun in the sky have its origin in several factors: the annual translational motion of Earth around the sun; Earth's daily rotation around its axis; and, maybe most important, the inclination of this axis with respect to ecliptic. The resulting motion of the sun in a horizon system can be seen in Figure I-1 (a).

As a result, considering that solar rays as coming parallel, the shadows casted by a vertical gnomon at those extreme positions make a triangle as the one shown in Figure I-1 (b). If we call α and β the elevations at noon of the sun above the horizon at the solstices (winter and summer, respectively) then, for a unitary gnomon, the *gnomonic factor* (***fg***) is

$$fg = \frac{1}{\tan\alpha} - \frac{1}{\tan\beta} \quad (I.1)$$

i.e.

$$fg = \frac{1}{\tan(90° - (\theta + \phi))} - \frac{1}{\tan(90° - (\theta - \phi))} \quad (I.2)$$
$$= \tan(\theta + \phi) - \tan(\theta - \phi)$$

Where θ is the Latitude at the observation site and ϕ is the solar declination defining the Tropics or obliquity of ecliptic.

Now, in the case of *platonic gnomonic factor* (***fgp***), defined as the difference between the winter solstice shadow and the shadow of the day when sun has a 45° elevation (*gnomon day* (***gd***)), we have

$$fgp = \frac{1}{\tan\alpha} - \frac{1}{\tan 45°} \quad (I.3)$$
$$= \tan(\theta + \phi) - 1$$

Using the expression for the tangent of the sum of angles we have

$$fgp = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} - 1$$
$$= \frac{\tan\theta(1 + \tan\phi) + \tan\phi - 1}{1 - \tan\theta\tan\phi} = \Delta \quad (I.4)$$

With the aid of this expression, we can obtain the Latitude of the site from the knowledge of the maximum declination of the sun (which depends, due to the obliquity of ecliptic, on the year we are considering) and the *platonic gnomonic factor* (renamed to Δ) which could be observed directly from the already named shadows.

$$\tan\theta = \frac{\Delta + 1 - \tan\phi}{1 + (\Delta + 1)\tan\phi} \quad (I.5)$$

i.e.

$$\theta = \tan^{-1}\left(\frac{\Delta + 1 - \tan\phi}{1 + (\Delta + 1)\tan\phi}\right) \quad (I.6)$$

Table I-1 Latitudes where the *Plato's Triangle* and *Sacred Triangle* were formed for different times.

Table I-1 Latitudes for different *fgp*'s values[a]			
Declination	Year[b]	"1/3"	"1/1"
23.5	1500	29.630	39.935
23.6	800	29.530	39.835
23.7	-20	29.430	39.735
23.8	-890	29.330	39.635
23.9	-1750	29.230	39.535
24.0	-2700	29.130	39.435
24.1	-3970	29.030	39.335
[a] All values are in Degrees (°) [b] minus sign means BC			

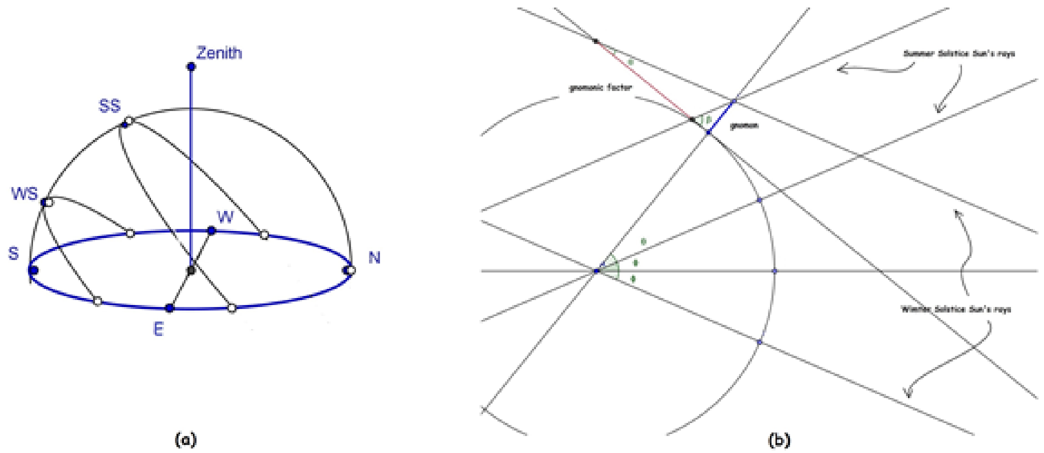

Figure I-1. Elements for the gnomonic factor (fg): (a) Sun's trajectories in a Horizon System at solstices (winter (WS) and summer (SS)); (b) Schematic view of the factor, including solstices' rays and shadows of a unitary gnomon; θ is the Latitude of the site and □represents the maximum solar declination (defining the Tropic of Cancer and Tropic of Capricorn); solar altitude above the Horizon is □ (□) for winter (summer) solstice.

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