

Trom Mathematical Logic to Quantum Logic: The Dilemma and Solution of the Logical Methodology of the Ideal Language School

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ARTICLE INFO	ABSTRACT
Received: 15 August 2024 Accepted: 01 September 2024	Mathematical logic, as the foundational tool of the ideal language school, seeks to eliminate the ambiguity and vagueness inherent in natural language through formalized symbolic systems and rigorous reasoning rules, striving for precise philosophical analysis. However, in practical applications, it encounters limitations such as formal isolation, static rigidity, and linear unidimensionality, which hinder its ability to address dynamic changes and complex, multidimensional relationships. Quantum logic, as a pioneering logical methodology, transcends these limitations with its core characteristics: contextual relevance, dynamic evolution, and nonlinear multidimensionality. It emphasizes the integration of logical expressions with specific contexts, adapts to changing systems, and adeptly manages multidimensional complexity. Quantum logic thus offers an innovative tool and theoretical foundation for formal language logic systems, potentially revitalizing the ideal language school. Keywords: Ideal Language School; Mathematical Logic; Quantum Logic; Logical Methodology;
	Contextual Relevance; Dynamic Systems; Multidimensional Complexity

1. INTRODUCTION

The ideal language school emerged from the analytic philosophy movement in the early 20th century, aiming to clarify philosophical problems by constructing precise and unambiguous formal languages. This approach sought to resolve traditional philosophical perplexities by eliminating the ambiguity and misuse of language. Mathematical logic serves as the core tool and method of this school, profoundly impacting philosophy, mathematics, and computer science throughout the 20th century. Pioneers such as Gottlob Frege, Bertrand Russell, and Rudolf Carnap dedicated themselves to unveiling the deep structure of language through logical analysis and formalization. Frege's *Begriffsschrift* (Concept Script, 1879) marked the inception of mathematical logic, while Russell's collaboration with Alfred North Whitehead on *Principia Mathematica* (1910-1913) stands as a monumental achievement of the ideal language school. Mathematical logic not only redefined philosophical inquiry but also provided new perspectives for the study of mathematical foundations and significantly influenced the development of modern information technology.

However, the rapid advancement of big data and artificial intelligence technologies has catalyzed an unprecedented shift in human cognition and information processing paradigms. Big data platforms now store and process vast amounts of information, uncovering potential patterns and regularities through complex algorithms. Artificial intelligence has achieved remarkable breakthroughs in mimicking and even surpassing human cognition. Consequently, the philosophy of language faces new expectations and challenges. While mathematical logic excels in formalization and precision, it struggles with nonlinear, multidimensional, and dynamically changing information. The complexity of modern information demands not only traditional logical tools and static models but also flexible, dynamic, and context-sensitive logical methods. For instance, advances in Natural Language Processing (NLP) have enabled significant achievements in machine understanding and generating natural language, largely relying on statistical learning, context recognition, and probabilistic reasoning rather than traditional mathematical logic. This necessitates innovation in logical tools and methodologies to meet the demands of big data and artificial intelligence. The limitations of mathematical logic are evident in its static nature and binary logic framework, which restrict its ability to describe dynamic changes and multidimensional complex systems. Therefore, modern intelligent environments require information

systems that can adapt and evolve to handle uncertainty and ambiguity.

Quantum logic, as an emerging logical methodology, offers a promising research path for the philosophy of language with its dynamic evolution and contextual relevance. It enables more effective engagement with complex real-world problems. In this context, the introduction of quantum logic represents not only an extension of mathematical logic but also a profound transformation of the research paradigm in the philosophy of language. This study aims to explore how quantum logic can overcome the limitations of mathematical logic in dealing with complex dynamic information systems, thereby providing innovative logical tools and methodological support for the era of big data and artificial intelligence. This endeavor represents both an innovation in research methods for the philosophy of language and a deepening and expansion of its theoretical foundations.

2. LIMITATIONS AND CHALLENGES OF MATHEMATICAL LOGIC

Mathematical logic, as a discipline dedicated to exploring formal systems and logical reasoning, is intricately linked to the theories of the ideal language school, serving as one of its foundational pillars. The primary objective of the ideal language school is to construct a rigorous formal language that can overcome the inherent ambiguity and vagueness of natural language, with mathematical logic providing the essential methodological framework to achieve this goal. The formalization methods of mathematical logic aim to uncover the laws governing human thought, marking a significant revolution in the field of logic (Feferman, 1984; Wang Yin, 2023).

The origins of mathematical logic can be traced back to the late 19th century, with Gottlob Frege as a pioneering figure. His work, *Begriffsschrift*, is considered the inception of modern formal logic, laying the groundwork for the symbolization and systematization of mathematical logic. By introducing predicate logic, Frege significantly enhanced the rigor and clarity of mathematical propositions and reasoning rules. His contributions provided a foundation for Bertrand Russell and Alfred North Whitehead, who further advanced these ideas in their attempt to establish a solid foundation for mathematics through formal systems (Russell & Whitehead, 1910-1913).

However, the advent of Kurt Gödel's incompleteness theorems revealed the inherent limitations of formal systems, demonstrating that within any sufficiently complex system, there exist propositions that cannot be proven or disproven using the system's internal rules (Gödel, 1931). This discovery was a profound revelation for the ideal language school, prompting a reassessment of the capabilities and boundaries of formal languages and logical systems. Simultaneously, Gödel's incompleteness theorems spurred the application and development of mathematical logic across broader scientific and technological domains (Feferman, 1984). As a result, mathematical logic shifted its focus from solely seeking complete and consistent systems to exploring the properties, models, and computational capabilities of formal systems. This shift gave rise to new research directions, including recursion theory, model theory, and proof theory.

By introducing rigorous symbolic systems and logical rules, mathematical logic enables the precise expression of mathematical propositions and reasoning processes. This formalization method not only eliminates the ambiguity of natural language but also enhances the transparency and verifiability of logical reasoning (Boolos & Jeffrey, 1980). Nevertheless, mathematical logic is not without its limitations. Beyond the intrinsic constraints highlighted by Gödel's incompleteness theorems, mathematical logic encounters significant challenges when addressing the complexity and dynamism of the real world (Feferman, 1984).

2.1 Formal Isolation

Mathematical logic, as a highly formalized logical system, centers on defining logical expressions and inference rules through symbolic and formal means. While this approach provides high precision and consistency in theory, it inevitably leads to the issue of formal isolation. Formal isolation refers to the emphasis on strict definitions and symbolic manipulation of logical expressions within mathematical logic's formal systems, independent of specific contexts and practical applications. This isolation renders the logical system overly abstract and detached from reality when addressing complex real-world problems.

On one hand, the process of symbolization and formalization in mathematical logic involves a high level of abstraction from the real world. For example, in propositional logic, symbols such as P and Q represent propositions, combined with logical connectives like \land , \lor , and \neg . Although this symbolization simplifies logical reasoning, it overlooks specific contexts and practical situations. On the other hand, the inference rules and logical axioms of mathematical logic are rigid and do not adapt to external conditions and contexts. For instance, the logical axiom $P \rightarrow (Q \rightarrow P)$ always holds, regardless of what P and Q specifically represent. This rigidity and lack of context sensitivity make mathematical logic insufficiently adaptable to real-world scenarios (Haack, 1978:123).

The root cause of formal isolation lies in the formal systems of mathematical logic, which overly emphasize symbolization and formalization, neglecting the complexity of specific contexts and practical applications. This high level of abstraction and independence makes mathematical logic inadequate when addressing complex real-world problems, falling short in coping with the ever-changing and intricate nature of reality. For example, in natural language processing, mathematical logic often struggles to handle the ambiguity and vagueness inherent in language, a significant issue in the philosophy of language (Frege, 1879). The ambiguity and vagueness of natural language systems pose substantial challenges, and the symbolic and formal characteristics of mathematical logic make it difficult to effectively address these issues.

Furthermore, mathematical logic faces new challenges in the era of big data and artificial intelligence, especially in processing and understanding large volumes of natural language data. These data are filled with complex contextual

dependencies and dynamic changes. Due to its formal isolation, the formal systems of mathematical logic find it difficult to meet these demands. This formal isolation limits the performance of mathematical logic in handling dynamically changing systems, thereby affecting its application in complex fields such as economic systems and ecological systems. In such cases, the rigidity and lack of adaptability of mathematical logic systems become increasingly apparent, further highlighting the issue of formal isolation.

For example, in medical diagnostic systems, the issue of formal isolation is particularly evident. Suppose we have a medical diagnostic system used to determine whether a patient requires further examination. We define the following propositions:

Proposition P(x): Patient x has a fever;

Proposition Q(x): Patient x has a cough;

Proposition I(x): Patient x needs isolation.

In mathematical logic, we can represent the diagnostic rule with the following logical expression: $\forall x(P(x) \land Q(x) \rightarrow I(x))$. This means "For all patients x, if patient x has a fever and a cough, then patient x needs isolation." While this process of symbolization simplifies the complexity of logical reasoning, it overlooks specific contexts and practical situations. For instance, whether a patient needs isolation may also depend on other factors such as the patient's age, medical history, and the current epidemiological situation.

To better adapt to practical situations, we can introduce more propositions and conditions. For example:

Proposition A(x): Patient x is older than 60 years;

Proposition H(x): Patient x has a medical history;

Proposition E: It is currently flu season.

The improved logical expression would be: $\forall x((P(x) \land Q(x) \land (A(x) \lor H(x) \lor E)) \rightarrow I(x))$. This means "For all patients x, if patient x has a fever and a cough, and patient x is older than 60 years or has a medical history or it is currently flu season, then patient x needs isolation." Even though the improved logical expression introduces more conditions, it still suffers from the problem of formal isolation, meaning it struggles to cope with the complexity and dynamic nature of real-world situations. It is necessary to comprehensively consider contextual factors and practical application scenarios to compensate for the limitations of formal systems in mathematical logic when dealing with complex real-world problems, thereby better meeting the demands of the modern era of big data and artificial intelligence.

2.2 Static Rigidity

Static rigidity is a fundamental characteristic of mathematical logic, referring to the static and immutable nature of its formulas and inference rules. These elements express fixed truth-value relationships and definite logical inference rules. While this rigidity ensures the precision and consistency of logical reasoning, it also makes logical systems appear inflexible when dealing with dynamically changing systems. The static nature of mathematical logic makes it difficult to capture the dynamic changes of real-world phenomena, limiting its adaptability in addressing complex and variable situations.

The static rigidity of mathematical logic primarily stems from its formal system design. The inference rules and logical axioms are based on strictly defined structures. For example, propositional logic and first-order logic in classical logic rely on fixed truth tables and definite inference rules. The truth tables in propositional logic define the outcomes of each proposition under different truth-value combinations, while the inference rules specify how to derive new propositions from known ones. This fixed nature renders mathematical logic inadequate when facing dynamically changing systems.

Consider the logic of autonomous vehicles as an example. Self-driving cars must make decisions in complex and dynamic environments, such as dealing with constantly changing traffic conditions, pedestrian behavior, and road conditions. Traditional logical systems struggle to handle these dynamic changes, potentially leading to incorrect decisions. For instance:

Premise 1: $J \rightarrow K$ (If there is an obstacle ahead, brake);

Premise 2: $M \rightarrow N$ (If a pedestrian is crossing the road, slow down).

Conclusion: $(J \rightarrow K) \land (M \rightarrow N)$.

The resulting logical expression states: "If there is an obstacle ahead, the car should brake; if a pedestrian is crossing the road, the car should slow down." Such expressions are too rigid in practical applications and fail to fully address the complexities and dynamics of environmental changes. For obstacles, a suddenly appearing obstacle might require different handling strategies (such as steering around instead of braking), but static logical rules cannot flexibly adjust. For unexpected pedestrian behaviors, such as sudden acceleration or abrupt stopping, a simple decision to slow down is insufficient. The ever-changing conditions of real-time traffic and unknown situations on the road demand that the logic system can quickly respond and adjust. However, mathematical logic systems cannot immediately update and dynamically adjust inference rules, impacting the safety and reliability of autonomous driving (Thrun, 2010).

Static rigidity primarily arises from the inherent fixed nature of the rules, which remain unchanged throughout the reasoning process and do not adjust according to environmental changes. This fixed nature makes it difficult for the system to cope with dynamic environments. Additionally, there is a lack of adaptability; mathematical logic systems cannot dynamically adjust their rules and decisions based on real-time data and environmental changes. In the logical expressions for autonomous driving, if a pedestrian suddenly changes behavior, the logical system cannot instantly update its rules to respond; it can only reason according to predefined rules. Even if a time variable (t) is added to mathematical logic, it does not change the static nature of the

reasoning; the time variable merely represents states at different points in time, not dynamically adjusting the rules. The logical system cannot modify its reasoning process based on the passage of time and environmental changes.

In theory, static rigidity provides mathematical logic with a high degree of precision and consistency, but in practical applications, especially when dealing with dynamically changing systems, its rigidity and inflexibility become prominent issues. Future developments need to more comprehensively consider dynamic factors and practical application scenarios to compensate for the limitations of mathematical logic in handling complex real-world problems, thereby better meeting the demands of the modern era of big data and artificial intelligence.

2.3 Linear Unidimensionality

The linear unidimensionality of mathematical logic refers to its logical expressions typically being linear and one-dimensional, making it difficult to express multidimensional and complex associations. This limitation restricts the ability of mathematical logic to handle multidimensional and complex systems, making it challenging to capture and describe intricate relationships. The basic unit of mathematical logic is the proposition, which is combined using logical connectives (such as "and", "or", "not", etc.) to form linear, one-dimensional logical expressions. On one hand, logical expressions are arranged in a certain linear order and processed sequentially, reflecting a high degree of determinacy and predictability in the logical reasoning process but limiting flexibility in handling multidimensional information. On the other hand, logical expressions in mathematical logic typically operate in a single dimension, making it difficult to simultaneously express information across multiple dimensions.

Consider a medical diagnosis system as an example. Let P represent the symptom of fever in a patient, Q represent the symptom of cough, and R represent the symptom of difficulty breathing. Additionally, let S represent influenza, T represent pneumonia, and U represent the common cold. Through logical expressions, the following diagnostic rules can be described:

 $P \land Q \rightarrow S$ indicates that if a patient has both fever and cough symptoms, they may have influenza.

 $P \land R \rightarrow T$ indicates that if a patient has both fever and difficulty breathing symptoms, they may have pneumonia.

 $\neg P \land Q \rightarrow U$ indicates that if a patient does not have a fever but has a cough symptom, they may have the common cold.

These logical expressions can be verified for their correctness using truth tables:

Ρ	Q	R	$P \wedge Q o S$	$P \wedge R o T$	$ eg P \wedge Q ightarrow U$
Т	Т	Т	Т	Т	F
Т	Т	F	Т	F	F
Т	F	Т	F	Т	F
Т	F	F	F	F	F
F	Т	Т	F	F	Т
F	Т	F	F	F	Т
F	F	Т	F	F	F
F	F	F	F	F	F

Although logical expressions and truth tables can clearly demonstrate the logical relationships of the diagnostic system under different combinations of symptoms, their linear and one-dimensional structure limits the ability to handle multidimensional and complex systems.

For example, actual medical diagnosis not only relies on a few symptoms but also needs to consider various factors such as the patient's medical history, age, gender, lifestyle habits, and other signs. The relationships among these factors are multidimensional and dynamically changing, making it difficult for the linear expressions of mathematical logic to fully capture and describe these complex relationships. Thagard (2005:45) discusses how cognitive processes often require a more nuanced approach than what linear logic can provide, emphasizing the need for systems that can handle complex, interconnected information.

In actual medical diagnosis, doctors need to comprehensively consider multiple factors, and there may be complex interactions among these factors. For instance:

The patient's medical history may affect the severity of current symptoms.

The patient's age and gender may influence the probability of certain diseases.

The patient's lifestyle habits (such as smoking and drinking) may increase the risk of certain diseases.

The relationships among these factors are multidimensional and complex, making it difficult for the linear expressions of mathematical logic to fully describe these multidimensional and complex relationships. This is because they can only be arranged in a certain sequence and cannot simultaneously express the interrelationships across multiple dimensions. Pearl (2009:99)

highlights the importance of causal models in understanding such complex systems, where linear logic falls short in capturing the causal interdependencies.

Language and meaning are also multidimensional and complex, influenced by context and the user. The meaning of language depends not only on the symbols themselves but also on the user's intent, context, and background knowledge. The formal system of mathematical logic represents logical expressions and reasoning processes through linear symbols and rules, ignoring the multidimensional and complex nature of language and meaning. This linear structure makes mathematical logic insufficient when dealing with multidimensional and complex systems, making it difficult to fully capture and describe complex real-world relationships.

Therefore, the linear unidimensionality of mathematical logic has the following limitations when dealing with multidimensional and complex systems. Firstly, it is difficult to express multidimensional information. Information in actual systems is often multidimensional. The linear expressions of mathematical logic find it challenging to simultaneously express the complex relationships among these multidimensional pieces of information. Secondly, it is difficult to capture dynamic changes. Information in actual systems is often dynamically changing. The static expressions of mathematical logic make it hard to capture these dynamic changes, leading to potentially inaccurate or untimely reasoning results. Finally, managing intricate associations presents a significant challenge. In practical systems, information often exhibits multifaceted interconnections that linear expressions in mathematical logic struggle to fully encompass and articulate.

3. QUANTUM MECHANICS AND QUANTUM LOGIC

3.1 Quantum Mechanics

Quantum mechanics represents a revolutionary shift in physics that began in the early 20th century. It was initiated in 1900 when Max Planck proposed the quantum hypothesis to address the anomaly of blackbody radiation, suggesting that energy is quantized. This marked a departure from classical physics and laid the groundwork for quantum theory (Planck, 1901).

Albert Einstein furthered quantum theory with his photon hypothesis in 1905, explaining the photoelectric effect and supporting the dual nature of light as both a wave and particle (Einstein, 1905). The concept of wave-particle duality was solidified by Louis de Broglie in 1924, who proposed that microscopic particles exhibit both wave-like and particle-like properties (de Broglie, 1924). This was confirmed by the double-slit experiment, underscoring a radical shift in understanding the microscopic world.

Werner Heisenberg's Uncertainty Principle introduced in 1927 further challenged classical determinism by demonstrating inherent limitations in measuring a particle's position and momentum simultaneously (Heisenberg, 1927). This principle underscores the probabilistic nature of quantum mechanics.

Erwin Schrödinger introduced the wave function concept and developed the foundational Schrödinger equation in 1926, which describes quantum state evolution and probability densities (Schrödinger, 1926). This theoretical framework successfully explained many microscopic phenomena, including atomic spectra.

Quantum superposition and entanglement further define the unique aspects of quantum mechanics. Superposition allows particles to exist in multiple states, while entanglement links the states of particles regardless of distance, as highlighted by the EPR paradox (Einstein, Podolsky, & Rosen, 1935). Despite initial skepticism, experiments such as those by Alain Aspect in 1982 confirmed entanglement's reality (Aspect et al., 1982).

The measurement problem, discussed by John von Neumann (1955), involves wave function collapse upon measurement, differing from classical expectations. These issues have led to various interpretations, including the Copenhagen and many-worlds interpretations.

The practical applications of quantum mechanics are vast. Technologies like semiconductors, lasers, and quantum computers stem from its principles. Quantum computing exemplified by Google's Sycamore processor, which achieved 'quantum supremacy' (Arute et al., 2019), showcases its potential to solve complex problems beyond classical capabilities.

Quantum mechanics not only reshapes microscale understanding but also affects macroscale phenomena. Superconductivity, described by Bardeen, Cooper, and Schrieffer (1957), is a macroscale manifestation of quantum mechanics, where electron pairs exhibit coherent quantum behavior leading to zero electrical resistance.

Quantum entanglement applications in macroscopic systems further illustrate quantum mechanics' broad applicability. Quantum Key Distribution (QKD) utilizes entanglement to achieve secure communication, as demonstrated by China's "Micius" satellite achieving significant milestones in quantum communication (Yin et al., 2017).

Quantum measurement advances have led to innovations in quantum sensors and imaging technologies, enhancing precision in various fields (Degen, Reinhard, & Cappellaro, 2017). These applications show quantum mechanics' profound implication in scientific and philosophical domains by challenging classical views on determinism and reality.

In summary, quantum mechanics has revolutionized our understanding of physics across micro and macro scales. The principles of wave-particle duality, quantum superposition, entanglement, and the measurement problem have drastically impacted both theoretical exploration and technological advancement.

3.2 Quantum Logic

The relationship between quantum mechanics and quantum logic is an important topic in modern philosophy of science and logic research. Quantum mechanics not only provides us with a theoretical framework for understanding the microscopic world but also lays the foundation for quantum logic through its unique principles and laws. The formal logical expressions and logical operations of quantum mechanics make quantum logic an effective tool for describing and analyzing quantum phenomena. The basic principles, tenets, and laws of quantum mechanics are not only the foundation, premise, and basis for the formation of quantum logic but also, through the formal expression of quantum logic, allow these principles and laws to be presented more precisely and systematically.

Firstly, the principle of superposition and the uncertainty principle in quantum mechanics provide an important theoretical foundation for the formation of quantum logic. The superposition state in quantum mechanics means that a quantum system can exist in multiple states simultaneously, a characteristic that cannot be described by classical logic. Classical logic is based on determinism and the law of the excluded middle, whereas quantum logic requires the introduction of new logical operations to handle superposition states. For example, Birkhoff and von Neumann's pioneering work in 1936 laid the foundation for quantum logic by introducing a lattice-theoretical framework that describes the logical structure of quantum mechanics, highlighting the differences from classical logic and emphasizing the role of superposition and coherence in quantum states (Birkhoff & von Neumann, 1936). These new logical operations not only reflect the unique properties of quantum mechanics but also provide theoretical support for quantum computing and quantum information processing. Specifically, a quantum state $|\psi\rangle$ can be expressed as a linear combination of multiple basis states $|\phi_i\rangle$, that is, $|\psi\rangle = \sum i c_i |\phi_i\rangle$, where c_i are complex coefficients. This expression demonstrates the multidimensional superposition of quantum states.

Secondly, the expression of quantum entanglement in quantum logic further demonstrates the close relationship between quantum mechanics and quantum logic. Quantum entanglement refers to the quantum states of two or more particles being interconnected such that even when they are spatially separated, measuring the state of one particle will immediately affect the state of the other. This phenomenon cannot be explained by classical logic but can be described in quantum logic through non-locality and correlation. For example, the violation of Bell's inequalities and the demonstration of non-locality in entangled states highlight the unique causal relationships and correlations inherent in quantum mechanics (Bell, 1964). Quantum logic introduces new operations and rules that help conceptualize the unique principles of quantum mechanics, offering insights foundational to quantum information science. An entangled state of two quantum systems can be mathematically represented in Dirac notation as $|\psi\rangle = \alpha |x_1\rangle |y_1\rangle + \beta |x_2\rangle |y_2\rangle$, where α and β are complex coefficients, $|x_1\rangle$ and $|x_2\rangle$ are basis states of system X, and $|y_1\rangle$ and $|y_2\rangle$ are basis states of system Y. This expression highlights the non-local correlations that define quantum entanglement, an essential feature for applications in quantum information science.

Additionally, the expression of quantum measurement theory and the wave function collapse principle in quantum logic is of significant importance. In quantum mechanics, the measurement process causes the wave function to collapse from a superposition to a definite state, a phenomenon that quantum logic formalizes through measurement and projection operators. For instance, the projection operator P can represent the probability distribution of measurement results and the wave function collapse process, expressed as $P|\psi\rangle = |\psi'\rangle$, where $|\psi'\rangle$ is the post-measurement quantum state (von Neumann, 1955). This formalism not only elucidates the essence of quantum measurement but also underpins theoretical frameworks for measurement and error correction in quantum computing and quantum information processing. By providing a structured approach to handling quantum states, these principles are crucial for the development of reliable quantum technologies.

The scientific validity, rationality, practicality, and operability of quantum logic have been widely verified in quantum computing and quantum information science. Quantum computers, leveraging the principles and operational rules of quantum logic, can outperform classical computers in specific computational tasks. For instance, Shor's algorithm, developed by Peter Shor in 1994, efficiently factors large integers, posing a significant challenge to classical cryptographic systems (Shor, 1994). Similarly, Grover's algorithm, introduced by Lov Grover in 1996, provides a quadratic speedup for unstructured database searches compared to classical algorithms (Grover, 1996). The success of these algorithms not only validates the scientific validity and rationality of quantum logic but also showcases its immense potential in practical applications, such as cryptography and data retrieval. Furthermore, ongoing advancements in quantum error correction and fault-tolerant quantum computing continue to demonstrate the operability and scalability of quantum logic in real-world applications (Nielsen & Chuang, 2002).

In summary, the principles and laws of quantum mechanics, through formal logical expressions and logical operations, provide a solid theoretical foundation for the formation of quantum logic. Quantum logic is not only a formal expression and generalization of the principles, theories, tenets, and laws of quantum mechanics but also demonstrates its scientific validity, rationality, practicality, and operability in quantum computing and quantum information science. Through rich case studies and specific applications, we can see the important status and wide application of quantum logic in modern science and technology.

3.3 Quantum Logic from the Perspective of Philosophy of Language

The development of quantum logic from the perspective of the philosophy of language begins with addressing the limitations inherent in classical mathematical logic. While mathematical logic excels in formalization and precision, it is often constrained by formal isolation, static rigidity, and linear unidimensionality, as previously discussed. These limitations hinder its effectiveness in modeling the dynamic and context-dependent nature of language and reality. Quantum logic emerges as a paradigm that transcends these constraints, offering a more adaptable and dynamic logical framework. It aligns more closely with the complexities of natural language and the nuanced interrelations observed in quantum phenomena. By incorporating principles such as contextuality and non-linearity, quantum logic provides a robust theoretical foundation for understanding and analyzing linguistic structures and meanings in ways that classical logic cannot. This approach is supported by the works of scholars like Birkhoff and von Neumann, who initially proposed quantum logic as a means to reconcile the peculiarities of quantum mechanics with logical reasoning (Birkhoff & von Neumann, 1936). Their pioneering work laid the groundwork for subsequent explorations

into the philosophical implications of quantum logic, as seen in more recent studies on its applications in linguistics and cognitive science (Aerts & Gabora, 2005).

The generation of quantum logic can be achieved by introducing fundamental concepts such as superposition, entanglement, and the temporal evolution of quantum states. The principle of superposition allows for the representation of multiple possibilities within logical expressions, reflecting the inherent probabilistic nature of quantum systems. Quantum entanglement, a phenomenon where particles become interconnected in such a way that the state of one cannot be described independently of the state of the other, provides a framework for describing non-local correlations between systems. This challenges classical notions of locality and separability, as initially discussed by Einstein, Podolsky, and Rosen (1935) and later experimentally confirmed by Bell's theorem (Bell, 1964). Temporal evolution operators, such as unitary transformations, enable quantum logic to dynamically describe the changes in systems over time, offering a more comprehensive understanding of system dynamics. By leveraging these fundamental concepts of quantum mechanics, quantum logic constructs new logical expressions that transcend the limitations of traditional mathematical logic, providing a more flexible and dynamic framework for reasoning about complex systems (Nielsen & Chuang, 2000). This approach not only enhances our theoretical understanding but also has practical implications for quantum computing and information processing, where these principles are actively applied.

Quantum logic from the perspective of the philosophy of language can be realized by integrating key concepts such as superposition, entanglement, and the temporal evolution of quantum states. The principle of superposition, which allows quantum systems to exist in multiple states simultaneously, introduces a framework for expressing multiple possibilities within logical expressions. This aligns with the linguistic concept of ambiguity and polysemy, where words or phrases can have multiple meanings depending on context. Quantum entanglement, a phenomenon where particles become interconnected such that the state of one cannot be described independently of the state of the other, provides a method for describing non-local correlations between systems. This challenges classical notions of locality and separability, offering a new perspective on the interconnectedness of meaning in language. Temporal evolution operators, such as unitary transformations, enable quantum logic to dynamically describe changes in systems over time, reflecting the fluid and evolving nature of language and meaning. By leveraging these fundamental concepts of quantum mechanics, quantum logic constructs new logical expressions that transcend the limitations of traditional mathematical logic, offering a more flexible and dynamic framework for reasoning about complex linguistic structures and their meanings. This approach not only enhances our theoretical understanding but also has practical implications for fields such as computational linguistics and artificial intelligence, where these principles can be applied to model and process natural language more effectively.

Consider a simple quantum system with an initial state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where $|0\rangle$ and $|1\rangle$ are the orthonormal basis states of the system, and α and β are complex coefficients satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. This quantum state can be conceptualized as a logical proposition P, indicating that the system is in state $|\psi\rangle$. Unlike traditional mathematical logic, where a proposition is strictly binary—either true or false—quantum logic allows for a probabilistic interpretation of truth values. The truth value of proposition P is represented by a probability distribution, specifically $|\alpha|^2$ and $|\beta|^2$, which denote the likelihood of the system being found in state $|0\rangle$ or $|1\rangle$, respectively. This probabilistic representation of the superposition state addresses the formal isolation of mathematical logic by enabling the simultaneous description of multiple potential states, reflecting the inherent uncertainty and complexity of quantum systems (Birkhoff & von Neumann, 1936).

Furthermore, consider the entangled state of two qubits $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. This entangled state can be represented as a composite proposition Q, signifying that the two qubits are in the entangled state $|\psi\rangle$. In classical logic, composite propositions are typically constructed through logical operations such as AND, OR, and NOT. However, in quantum logic, the entangled state Q embodies a non-local correlation that defies decomposition into simpler logical operations. This is due to the intrinsic quantum property of entanglement, where the state of each qubit cannot be independently described without reference to the other, a phenomenon that Einstein famously referred to as "spooky action at a distance" (Einstein, Podolsky, & Rosen, 1935). This representation transcends the static rigidity of mathematical logic by capturing the dynamic and holistic correlations between quantum systems, which are essential for understanding phenomena such as quantum teleportation and superdense coding (Nielsen & Chuang, 2002).

Additionally, quantum logic can effectively describe the dynamic changes of a system through the temporal evolution operator U(t). Consider a quantum state $|\psi(0)\rangle$ at the initial time t=0. By applying the temporal evolution operator U(t), which is typically a unitary operator, we can determine the state of the system at a later time t as $|\psi(t)\rangle = U(t)|\psi(0)\rangle$. This process of temporal evolution can be conceptualized as a dynamic proposition R, indicating that the system transitions from the initial state $|\psi(0)\rangle$ to the state $|\psi(t)\rangle$. This representation of dynamic propositions transcends the linear unidimensionality of traditional mathematical logic by capturing the continuous and complex evolution of quantum systems over time, reflecting the inherently dynamic nature of quantum mechanics (Sakurai & Napolitano, 2017).

Through these fundamental expressions of quantum logic, we can construct a logical framework that surpasses the limitations of classical mathematical logic. In the realm of quantum computing, quantum logic gates, such as the Hadamard gate and the CNOT gate, are employed to manipulate quantum states, facilitating the execution of complex quantum algorithms. For instance, Shor's algorithm, a groundbreaking quantum algorithm, performs the prime factorization of large integers by orchestrating a series of operations involving quantum logic gates. This quantum computing process leverages not only the principles of superposition and entanglement but also necessitates the use of temporal evolution operators to accurately describe the dynamic transformations of quantum states throughout the computation (Nielsen & Chuang, 2002). This integration of quantum logic into computing exemplifies its potential to revolutionize computational paradigms, offering exponential speedups for specific problems compared to classical approaches..

In summary, from the perspective of the ideal language school within the philosophy of language, quantum logic effectively overcomes the limitations of mathematical logic, such as formal isolation, static rigidity, and linear unidimensionality. It offers a more flexible and dynamic logical framework that aligns with the ideal language school's goal of precisely analyzing and resolving philosophical issues. Quantum logic enhances our ability to describe interactions and correlations in complex systems and dynamically depict their evolution over time. This advancement not only addresses the challenges faced by traditional logical methods but also provides a solid theoretical foundation for the development of quantum computing and quantum information science, heralding a new era for the ideal language school.

4. THE ADVANTAGES AND POTENTIAL OF QUANTUM LOGIC

In his seminal work *Tractatus Logico-Philosophicus*, Wittgenstein introduced the picture theory, which has become a foundational concept for understanding the relationship between logic and reality. A central proposition of the picture theory is that "a picture represents a possible situation in logical space." Wittgenstein posited that a picture corresponds to a state of affairs in the real world through its logical form, thereby depicting reality. The meaning of a picture is determined by its consistency or inconsistency with reality, allowing it to be judged as true or false (Wittgenstein, 1922). This proposition highlights the logical relationship between pictures and reality, serving as a bridge between abstract logical structures and tangible real-world scenarios.

However, when we attempt to apply mathematical logic to this framework, we encounter significant challenges. Mathematical logic, with its reliance on precise and often rigid structures, excels in scenarios where relationships are straightforward and well-defined. Yet, it often struggles to adequately capture the complexity inherent in real-world systems, which are frequently characterized by intricate and multidimensional relationships. This limitation becomes evident when we analyze Wittgenstein's propositions in the context of complex systems, where the simplistic nature of traditional logical expressions proves insufficient.

By examining Wittgenstein's propositions, we can better understand these limitations. For instance, mathematical logic typically employs simple predicates and quantifiers to represent the correspondence between pictures and reality. An expression like $\forall x (P(x) \rightarrow \exists y (R(y) \land M(x, y)))$, suggests that every element in the picture corresponds to some object in reality. However, this approach is overly simplistic when addressing complex systems, making it challenging to manage multiple correspondences and dynamic changes. As Quine noted, while logical concepts are useful in certain contexts, their utility diminishes when dealing with more complex situations, such as the individuation of propositions (Quine, 1986). He critiqued the doctrine of propositions, arguing that they are merely shadows of sentences and offer no more information than the sentences themselves. This underscores the limitations of mathematical logic in addressing complex systems and multidimensional relationships.

In contrast, quantum logic offers a more flexible and dynamic means of expression through quantum states and entangled states. Quantum states, such as $|\psi\rangle = \sum \alpha_i |element_i\rangle$ and $|\psi\rangle = \sum \alpha_i |object_j\rangle$, along with entangled states $|\psi\rangle = \sum \alpha_{ij} |element_i\rangle$ object $_j\rangle$, naturally describe the complex relationships and dynamic changes between elements and objects. For example, $|\alpha_{ij}|^2$ represents the strength of the correlation between element i and object j, enabling more effective handling of multiple correspondences and dynamic relationships that evolve over time (Nielsen & Chuang, 2002). Quantum logic not only captures multidimensional relationships in complex systems but also adeptly manages the dynamic evolution and contextual correlations of situations through the superposition and entanglement of quantum states.

In this section, we will examine three core propositions from Wittgenstein's *Tractatus Logico-Philosophicus* concerning the picture theory to underscore the advantages of quantum logic over mathematical logic. These propositions are Proposition 2.15, Proposition 33, and Proposition 34 (Wittgenstein, 1922). By contrasting these propositions, we aim to demonstrate how quantum logic surpasses mathematical logic in expressing complex, dynamic, and multidimensional relationships.

4.1 Contextual Relevance

In discussing "Proposition 2.15: The elements of the picture correspond to the objects of reality", mathematical logic often employs predicates and quantifiers to express this correspondence. For instance, let P(x) denote that x is an element of the picture, R(y) denote that y is an object in reality, and M(x,y) denote that x corresponds to y. Through the expression $\forall x(P(x) \rightarrow \exists y(R(y) \land M(x,y)))$, predicate logic can represent that every element in the picture corresponds to some object in reality. This logical framework is effective for simple one-to-one correspondence relationships but encounters difficulties with complex or multiple correspondences. For example, when an element in the picture may correspond to multiple real objects, or when this correspondence changes over time, predicate logic becomes inadequate.

To address these complexities, consider two quantum systems: one representing the elements in the picture X and the other representing the objects in reality Y. To express the intricate relationships between them, we use an entangled state:

$$|\Psi\rangle = \sum_{ij} a_{ij} |i\rangle_X |j\rangle_Y$$
 (4-1)

In equation (4-1), $|i\rangle_X$ represents the i-th element in the picture, $|j\rangle_Y$ represents the j-th object in reality, and α_{ij} is a complex coefficient representing the strength of the correlation between element i and object j. The state of the system $|\Psi\rangle$ is a superposition of all possible state combinations ((i,j), with each combination represented by the basis states $|i\rangle_X$ and $|j\rangle_Y$, and the joint probability amplitude of each combination given by the complex coefficient α_{ij} . This superposition state can simultaneously describe all possible states and their interrelationships of the two subsystems X and Y, thereby reflecting the overall state of the entire system.

The interpretation of the quantum logic expression is as follows:

Multiple Correspondences: The quantum state $|\Psi\rangle$ can naturally represent multiple correspondences. An image element i can correspond to multiple real objects j simultaneously because α_{ij} can have multiple non-zero terms.

Contextual Relevance: The superposition and entanglement features of quantum states allow them to express contextual relevance. For instance, each α_{ij} in $|\Psi\rangle$ can depend on contextual information, enabling the correspondence between image elements and real objects to adjust according to different contexts.

Through this analysis, we observe that quantum logic offers significant advantages in handling complex relationships between elements and objects. It can naturally describe dynamic changes, represent multiple correspondences in complex systems, and account for the influence of context on the relationships between elements and objects. Thus, quantum logic exhibits greater potential in describing and understanding complex systems.

4.2 Dynamic Evolution

In exploring Proposition 33: "Images portray reality by presenting the possibility of the existence and non-existence of states of affairs", mathematical logic typically employs a qualitative method to express the presence or absence of states of affairs. For example, the expression $\forall x(S(x) \rightarrow (E(x) \lor \neg E(x)))$ indicates that each state of affairs either exists or does not exist. That is, for every state of affairs x, if x is a state of affairs S(x), then x either exists E(x) or does not exist $\neg E(x)$. This binary logic overlooks the dynamic changes of states of affairs across different time points or contexts and cannot fully express the complexity of states of affairs (Kleene, 1971).

In contrast, quantum logic handles the existence and non-existence of states of affairs more flexibly through the principle of superposition of quantum states. The quantum logic formula for Proposition 33 is expressed as follows:

$$| \psi(t) \rangle = U(t)(\alpha | E \rangle + \beta | N \rangle)$$
 (4-2)

Assuming at the initial moment t=0, the quantum state $|\psi(0)\rangle$ represents the state of affairs:

$$| \psi(\mathbf{0}) \rangle = \alpha | \mathbf{E} \rangle + \beta | \mathbf{N} \rangle$$
 (4-3)

Here, $| E \rangle$ represents the state of affairs existing, $| N \rangle$ represents the state of affairs not existing, and α and β are complex coefficients that satisfy the normalization condition $| \alpha |^2 + | \beta |^2 = 1$.

Assuming that the quantum state evolves over time, described by the time evolution operator U(t), the quantum state at time *t* is expressed as:

$$\psi(t) = U(t) | \psi(0) \rangle$$
. (4-4)

According to the Schrödinger equation, the time evolution of a quantum state is described by a unitary operator, ensuring that the normalization of the state remains unchanged during the evolution.

Thus, it is evident that quantum logic can reflect the changes of states of affairs at different time points and contexts, which is consistent with the time evolution and contextual dependency of states in quantum mechanics. The evolution of a quantum state can change according to different external conditions (such as external fields, measurements, etc.), a characteristic that allows quantum logic to dynamically adapt to changes in both internal and external environments of a system, providing significant advantages in handling dynamic systems.

4.3 Non-linear Multidimensionality

Proposition 34: "The image represents a possible state of affairs in the logical space" is typically expressed in mathematical logic as $\exists x(P(x) \land L(x))$, indicating the existence of a possible state of affairs in the logical space. Here, P(x) denotes that x is a possible state of affairs, and L(x) denotes that x is in the logical space. While this expression is intuitive, its linear and unidimensional characteristics make it difficult to address the complexity of states of affairs across different dimensions and contexts. For example, in complex social networks, the interweaving and influence of multiple relationships are challenging to describe clearly through simple logical expressions.

To express the quantum logic of Proposition 34, we use quantum states and entangled states to represent the complex relationships of states of affairs across different dimensions and contexts. The quantum logic expression can be represented as:

$$|\Psi\rangle = \sum_{ij} a_{ij} |i\rangle |j\rangle$$
 (4-5)

The quantum state $|\psi\rangle$ represents the state of affairs, where $|i\rangle$ and $|j\rangle$ represent states of affairs in two different dimensions, and α_{ij} are complex coefficients indicating the strength of association between state of affairs i and j. The state of the system $|\psi\rangle$ is a superposition of all possible combinations of states of affairs (i, j), with each combination represented by basis states $|i\rangle$ and $|j\rangle$, and the strength of association for each combination given by the complex coefficient α_{ij} . This superposition state can simultaneously describe multiple states of affairs and their interrelationships, thereby reflecting the overall state of the system. This expression fully embodies the non-linear multidimensionality of quantum states, allowing for more flexible handling of the complex relationships of states of affairs across different dimensions and contexts.

Compared to mathematical logic, quantum logic possesses significant non-linear multidimensionality:

Multidimensional Expression: Quantum states can simultaneously represent states of affairs across multiple dimensions, with the relationships between these states being describable via entangled states. This multidimensional expression method can

more comprehensively reflect the multiple relationships within complex systems.

Non-linear Relationships: The superposition and entanglement properties of quantum states allow for the expression of non-linear relationships. Multiple states of affairs can coexist in a superposition state, and their interrelationships can be described through entangled states.

Contextual Association: The coefficients of a quantum state can depend on contextual information, enabling quantum logic to dynamically adjust the relationships between states of affairs based on different contexts.

Through quantum logic expressions, we can more flexibly and dynamically express the complex relationships of states of affairs across different dimensions and contexts. Quantum logic not only handles multidimensional relationships but also naturally expresses contextual associations, demonstrating significant advantages in handling complex systems. Compared to mathematical logic, quantum logic exhibits stronger non-linear multidimensionality, allowing for a more comprehensive expression of the complexity of states of affairs. This approach not only naturally describes the dynamic changes of states of affairs but also handles strong associations between states of affairs through entangled states.

5.CONCLUSION

This paper explores the challenges and solutions in the logical methodology of the ideal language school by comparatively analyzing mathematical logic and quantum logic. We have highlighted the advantages of quantum logic in handling complex systems and dynamic changes. While mathematical logic, as a primary tool, excels in formalization and precision, it faces limitations such as formal isolation, static rigidity, and linear unidimensionality in practical applications. These limitations hinder its ability to address the complexities and variability of real-world problems.

To overcome these challenges, we propose quantum logic as a novel logical methodology. Quantum logic is characterized by contextual relevance, dynamic evolution, and non-linear multidimensionality, enabling it to more flexibly manage multidimensional relationships and dynamic changes in complex systems. Through the analysis of three propositions of Wittgenstein's picture theory, we demonstrated the significant advantages of quantum logic in expressing complex relationships and dynamic changes the intricate relationships between elements and objects and manages the dynamic evolution and contextual association of states of affairs through the superposition and entanglement of quantum states.

In summary, the principles and laws of quantum mechanics provide a robust theoretical foundation for the development of quantum logic through formal logical expression and computational functions. Quantum logic not only encapsulates the principles, theories, and laws of quantum mechanics but also demonstrates its scientific, rational, practical, and operable nature in quantum computing and quantum information science. As an evolution and transcendence of mathematical logic, quantum logic holds the potential to rejuvenate the ideal language school.

Future research can further explore the application of quantum logic in various fields, fostering a paradigm shift in the philosophy of language and offering new tools and theoretical foundations for addressing more complex philosophical problems. Through this interdisciplinary research, we aspire to achieve breakthroughs across broader scientific and philosophical domains.

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